

Simplify the rational expression.

$$\frac{y^2 - 3y + 9}{y^3 + 27}$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$\frac{y^2 - 3y + 9}{y^3 + 27} = \frac{y^2 - 3y + 9}{(y + 3)(y^2 - 3y + 9)}$$

$$= \frac{\cancel{(y^2 - 3y + 9)}}{(y + 3)\cancel{(y^2 - 3y + 9)}} = \frac{1}{(y + 3)} \quad \text{whenever } y^2 - 3y + 9 \neq 0 \text{ and } y \neq -3.$$

$$\text{Note: } y^2 - 3y + 9 = 0 \text{ when } y = \frac{3 \pm \sqrt{9 - 4(1)(9)}}{2} = \frac{3 \pm \sqrt{-27}}{2} = \frac{3 \pm 3i}{2}$$